[a] $(\cos x)\,dy + (2y^2\cos x - \sin x)y\,dx = 0$ **FINAL ANSWER**: $y^2 = \frac{1}{4\sin x\cos x + C\cos^2 x}$ OR $y^2 = \frac{1}{2\sin 2x + C\cos^2 x}$

$\frac{dy}{dx} + (2y^2 - \tan x)y = 0 \Rightarrow \frac{dy}{dx} - (\tan x)y = -2y^3$ ⬅BERNOULLI

$v = y^{1-3} = y^{-2} \Rightarrow \frac{dv}{dx} = -2y^{-3}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}y^3\frac{dv}{dx}$

$-\frac{1}{2}y^3\frac{dv}{dx} - (\tan x)y = -2y^3 \Rightarrow \frac{dv}{dx} + (2\tan x)y^{-2} = 4 \Rightarrow \frac{dv}{dx} + (2\tan x)v = 4$ ⬅LINEAR

$\mu = e^{\int 2\tan x\,dx} = e^{-2\cos x} = \sec^2 x$

$(\sec^2 x)\frac{dv}{dx} + (2\sec^2 x\tan x)v = 4\sec^2 x$ **CHECKPOINT**: $\frac{d}{dx}\sec^2 x = 2\sec x\sec x\tan x = 2\sec^2 x\tan x$

$(\sec^2 x)v = 4\tan x + C \Rightarrow v = 4\sin x\cos x + C\cos^2 x \Rightarrow y^{-2} = 4\sin x\cos x + C\cos^2 x$

[b] $\frac{dr}{dt} = \frac{r^4 + t^4}{2rt^3}$ **FINAL ANSWER**: $r^2 = t^2 - \frac{t^2}{\ln|t| + C}$

$2rt^3\,dr - (r^4 + t^4)\,dt = 0$ ⬅HOMOGENEOUS since $2(kr)(kt)^3 = k^4(2rt^3)$ and $(kr)^4 + (kt)^4 = k^4(r^4 + t^4)$

$r = vt \Rightarrow 2vt^4(v\,dt + t\,dv) - (v^4t^4 + t^4)\,dt = 0 \Rightarrow 2v(v\,dt + t\,dv) - (v^4 + 1)\,dt = 0$

$\Rightarrow 2vt\,dv - (v^4 - 2v^2 + 1)\,dt = 0 \Rightarrow \frac{dv}{dt} = \frac{v^4 - 2v^2 + 1}{2v}\cdot\frac{1}{t}$ ⬅SEPARABLE

$\Rightarrow \int\frac{2v}{(v^2-1)^2}\,dv = \int\frac{1}{t}\,dt \Rightarrow \frac{1}{1-v^2} = \ln|t| + C \Rightarrow \frac{1}{1-(\frac{r}{t})^2} = \ln|t| + C \Rightarrow \frac{t^2}{t^2 - r^2} = \ln|t| + C$

[c] $\frac{dz}{dw} = \frac{4e^{2w}}{6e^{2w} - 3e^{5z}}$ **FINAL ANSWER**: $3e^{5z} + 4e^{2w} = Ke^{3z}$

$(3e^{5z} - 6e^{2w})\,dz + 4e^{2w}\,dw = 0$

$M = 3e^{5z} - 6e^{2w}, N = 4e^{2w} \Rightarrow M_w = -12e^{2w}, N_z = 0 \Rightarrow \frac{M_w - N_z}{N} = \frac{-12e^{2w}}{4e^{2w}} = -3$ ⬅FUNCTION OF ONLY z

$\mu = e^{\int -3\,dz} = e^{-3z}$

$(3e^{2z} - 6e^{-3z}e^{2w})\,dz + 4e^{-3z}e^{2w}\,dw = 0$ **CHECKPOINT**: $(3e^{2z} - 6e^{-3z}e^{2w})_w = -12e^{-3z}e^{2w} = (4e^{-3z}e^{2w})_z$

$f = \int(3e^{2z} - 6e^{-3z}e^{2w})\,dz = \frac{3}{2}e^{2z} + 2e^{-3z}e^{2w} + C(w)$

$f_w = 4e^{-3z}e^{2w} + C'(w) = 4e^{-3z}e^{2w} \Rightarrow C'(w) = 0$ **CHECKPOINT**: FUNCTION OF ONLY w

$C(w) = 0 \Rightarrow \frac{3}{2}e^{2z} + 2e^{-3z}e^{2w} = K$